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PHENOMENAL REPORT

Five types of anomalous perceptions created by the same mirror-reflection process

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Abstract

The real world and its mirror image are plane symmetric to each other with respect to a mirror surface, but mirror images are often perceived differently due to optical illusions. This paper shows that there are at least five types of different illusory perceptions of mirror images of pictures, although they arise from the same optical process. In all five illusions, we place a 2D picture of a 3D object horizontally, and see its image reflected by a vertical mirror. In some cases, the left and the right are reversed (left-right reversal illusion), sometimes horizontal planes change their heights (height reversal illusion), sometimes a lying object rises up (lying-standing illusion), sometimes the object turns upside down as if having done a somersault (somersault illusion), and sometimes the object is replaced by a rather different object (replacement illusion). This paper examines why these illusions occur from geometrical and psychological points of view.

Keywords: Picture; 3D interpretation; Depth perception; Left-right reversal illusion; Height reversal illusion; Lying-standing illusion; Somersault illusion; Replacement illusion.

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Mirror reflection is a purely optical process, and this process can be understood clearly from geometry. An object and its mirror image are plane symmetric to each other with respect to the mirror surface as the plane of symmetry, so that a right-handed system in the real world becomes a left-handed system in the mirror, for example.

However, the mirror image we perceive sometimes behaves counterintuitively because of optical illusions. A typical example is the ambiguous cylinder illusion, in which the mirror image of a 3D object appears to be quite different from the original object, and hence we feel that we are looking at something impossible (Sugihara, 2015a). The ambiguous cylinders have been extended in several directions so that the objects partially disappear in the mirror (Sugihara, 2016b), topological structures change in the mirror (Sugihara, 2018), and tiling patterns change to other tiling patterns in the mirror (Sugihara, 2020a). Nevertheless, these appearances might not be so surprising when considering that the visible parts of 3D objects change when the viewpoint is changed; in other words, when the object is reflected in the mirror, part of the object becomes occluded and other parts come into view.

When we see a 2D picture, on the other hand, the change of the viewpoint usually does not generate a drastic change in appearance. The whole part of the picture continues to be visible unless the viewpoint moves to the back side of the 2D plane that the picture is on, and the deformation of the picture obeys 2D projective-geometry rules. Still, there are some exceptions, which have two or more interpretations. The Necker cube is a picture of the wireframe of a cube, in which the front side and the back side flip in the viewer's mind (Gregory, 1970; Necker, 1832). The Mach book can be interpreted as an open book standing on a desk and as an open book faced down on a desk surface (Robinson, 1998). The Schröder staircase can be interpreted as a staircase looked down to from above and as a rear side of the staircase looked up

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to from below (Robinson, 1998). One interpretation flips to the other occasionally, but the two interpretations do not exist in the viewer's mind simultaneously. Sometimes the flip of the interpretation is triggered by a change in the way of looking at the picture such as by turning the picture upside down.

On the other hand, if we place a 2D picture horizontally in front of a vertical mirror and see both the picture and its mirror image, what we experience is a little different from the traditional ambiguous pictures. Indeed, we see the two appearances at the same time and we know that both the picture and the mirror are placed in the same world that we live in. In this situation, we sometimes encounter anomalous perceptions in which we feel that the mirror image cannot come from the original picture and hence that the object is impossible (Sugihara, 2020b). The author has found five types of such illusory perceptions. These are named as follows: (1) the 'left-right reversal' illusion in which the left and right of the object is reversed in the mirror, (2) the 'height reversal' illusion in which the order of flat areas from the lowest to the highest is reversed in the mirror, (3) the 'lying-standing' illusion in which a lying object rises up in the mirror, (4) the 'somersault' illusion in which the object turns upside down in the mirror as if it has performed a somersault in the space, and (5) the 'replacement' illusion in which the object is replaced with another object in the mirror.

In the following sections of this paper, we show examples of the above named five visual effects (Section 2), consider how each occurs from the point of view of geometry (Sections 3 to 7), and discuss possible factors by which each of these visual effects occurs in our perception (Section 8).

Examples of the five anomalous perceptions

Figs. 1, 2, 3, 4, and 5 present examples of the five types of anomalous perceptions. In each of the Figs, a 2D picture is placed horizontally on a desk surface, a planar mirror is placed vertically behind it in such a way that the mirror surface faces toward the viewer, and the scene is viewed in a slanted direction from above. The current classification of the five types of anomalous perceptions is based on the author's subjective interpretations.

Fig. 1 shows a visual effect in which the left and right parts of the object are reversed in the mirror. In panel (a), the object consists of a staircase rising from left to right, but in the mirror the staircase rises in the opposite direction, that is, from right to left. Moreover, the lower flat area, the side walls and the upper flat area are all flipped between the left and the right. Thus, we feel that the left and the right of the object are reversed in the mirror. In panel (b), the picture represents a room wherein the floor on the left and ceiling on the right are red, and a similar room can be seen in the mirror except that the floor on the right and ceiling on the left are red. Hence, we likewise feel that the left and right sides of the object are reversed in the mirror. These are examples of the left-right reversal illusion. Another example can be seen by videos (Sugihara 2020c).

Note that the pictures are not bounded by normal rectangular frames. The objects are drawn on a planar surface, and the surrounding backgrounds are cut off. Consequently, the boundaries of the pictures can have complicated shapes depending on what is drawn.

Fig. 2 shows a visual effect in which a convex surface becomes concave in the mirror. In panel (a), the surface shows a round hill, but in the mirror, it appears as a round hole. In panel (b), the top of the box protrudes upward, but in the mirror, it is dug downward. Thus, the object height is reversed in the mirror.

Fig. 3 shows a visual effect in which a lying object appears to have risen up in the mirror. In panel (a), a nut is placed on its side, but in the mirror, it is placed so that the flat side faces upward. In panel (b), a bundle of cylinders is placed horizontally, but in the mirror, it has risen up to stand vertically. Thus, the lying object stands in the mirror. Other examples can be seen by videos (Sugihara 2021).

(a) (b)

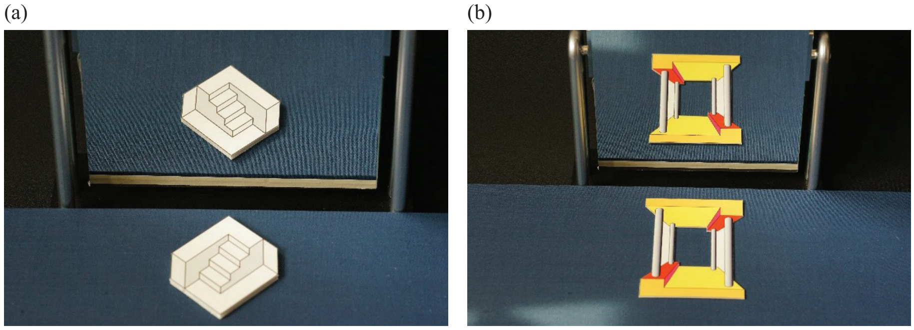

Fig. 1. Left-right reversing objects.

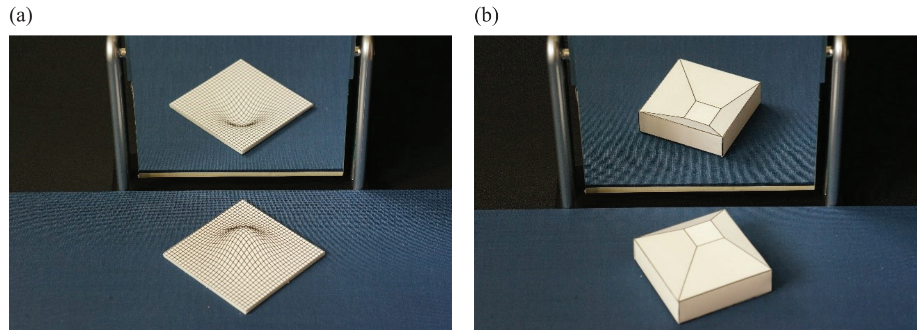

Fig. 2. Height-reversing objects.

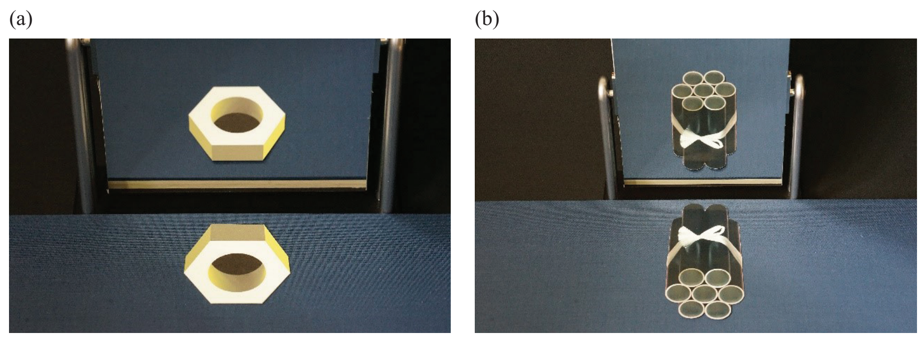

Fig. 3. Lying-standing illusion objects.

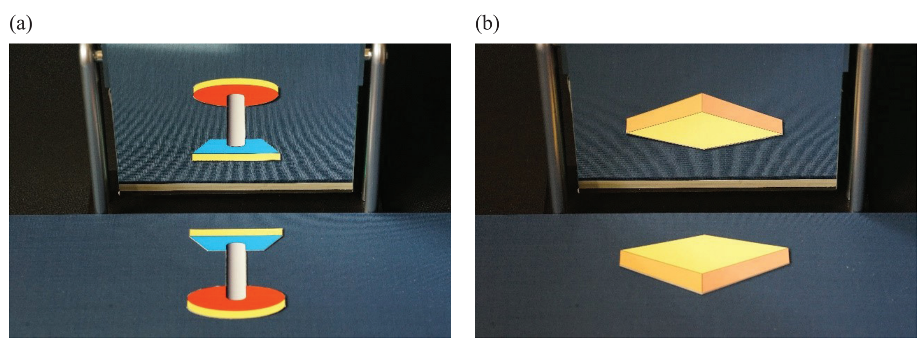

Fig. 4. Somersaulting objects.

Fig. 4 shows a visual effect in which an object turns upside down in the mirror as if having done a somersault. In panel (a), a square table having a round foot turns upside down in the mirror and as a result, the object changes to a round table with a square foot. In panel (b), the square board is placed on the table facing upward, but in the mirror, it faces downward. Thus, the object is upside down in the mirror.

Fig. 5 shows a visual effect in which the object in the mirror is quite different from the original, and so we feel as if the object is replaced with another. In both panel (a) and panel (b), the red cone is a real 3D object, while all the other parts are a 2D picture placed horizontally. In panel (a), a large upper flat area is connected to a narrow lower area by two staircases, but in the mirror the two staircases meet at a narrow upper area. We feel that these 3D structures are different from each other. In panel (b), a cone is placed on the top of a fan-shaped plate attached to a complicated wall, but the mirror image looks like another fan-shape plate attached to a different wall. It is likewise

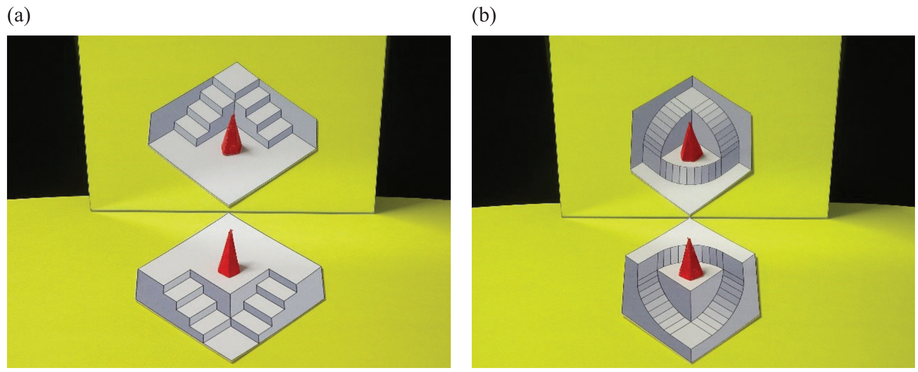

Fig. 5. Replaced objects.

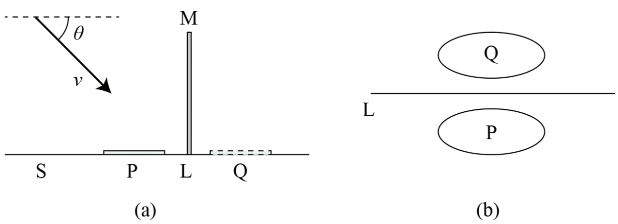

Fig. 6. The same optical process, which creates five different types of visual effects: (1) the side view of the scene; and (b) the scene seen along the viewing direction.

difficult to understand that this effect can just come from the mirror reflection.

We have seen five different types of visual effects created by a picture and its mirror image. Note that the optical process in all of these is the same. The situation is represented in Fig. 6. As shown in the side view in panel (a), we place a picture P horizontally on a desk surface S, place a vertical mirror M behind it, and see the scene from the viewing direction v, which is slanted downward by angle θ measured from the horizontal direction. We denote by L the line of intersection of the desk surface and the mirror surface, and denote by Q the mirror image of P. P and Q are plane symmetric with respect to the mirror surface. Panel (b) shows the appearance of the scene seen along the viewing direction v for the case $\theta = 45$ degrees. Because P and Q are plane symmetric and both of them are planar and horizontal, the appearance of P and that of Q in panel (b) should be line symmetric with respect to the line L. Indeed, all the images in Figs. 1, 2, 3, 4, and 5 admit these line-symmetry characteristics. Next, let us consider why this optical process creates different perceptual effects.

Left-right reversal

The picture used in Fig. 1a is presented in Fig. 7. This picture represents the 3D structure consisting of a staircase, side walls, and top and bottom flats. It has at least two remarkable properties.

First, the steps of the staircase are formed by rectangles instead of general parallelograms. The upper and lower flat areas are also formed by rectangular corners. These surfaces should be horizontal in the 3D space, and this property is consistently perceived if we place the picture horizontally and look at it downward from a slanted direction. The human brain has strong preference for rectangles to other angles when interpreting pictures as 3D structures (Perkins, 1972, 1973; Sugihara & Pinna, 2022), and can guess the rough orientation of the surface normal from the apparent shape and posture of a parallelogram. Because the rectangles are placed horizontally in Fig. 1a, we can naturally perceive the staircase with horizontal steps. In other words, we perceive a 3D structure instead of a 2D picture, which may make it difficult for us to recognise that the picture and its mirror image are line symmetric.

Secondly, the picture in Fig. 7 is point symmetric with respect to the centre point; if we rotate the picture by 180 degrees, the resulting picture coincides with the original picture. This means that if we place the picture horizontally and rotate it around the vertical axis by 180 degrees, the appearance of the picture returns to the original one (Sugihara, 2016a). The rotation around the vertical axis by 180 degrees is equivalent to seeing the picture from the opposite side by the same downward-looking angle, and this in turn is equivalent to reversing the left and the right sides and to seeing the mirror image. This is why the mirror image corresponds to the left-right reversed version of the original 3D structure.

The picture used in Fig. 1b is also point symmetric with respect to the centre. Therefore, by the same reasoning, the mirror image coincides with the left-right reversed version of the structure represented by the original picture. Note that this picture is drawn using rules of perspective projection, which may also strengthen the impression of a 3D structure rather than a 2D picture, and may make it difficult to recognise the line symmetry.

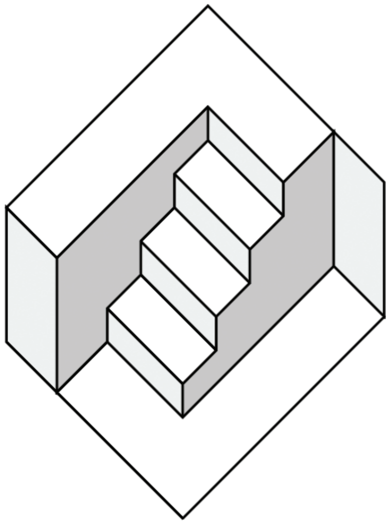

Fig. 7. Picture used in Fig. 1a.

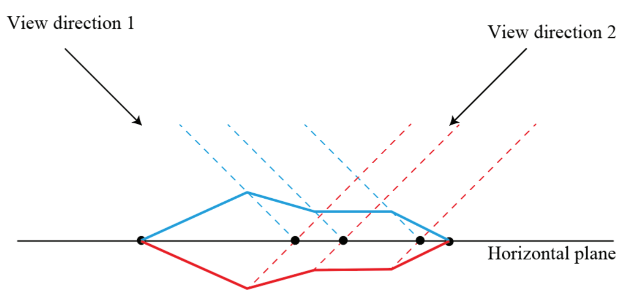

Fig. 8. Height-reversal property of a horizontally placed picture.

Height reversal

In this visual effect, a horizontally placed picture has the remarkable property that it gives two interpretations of 3D structures, one of which is obtained by reversal of the height of the other when it is seen from the opposite side with the same downward-looking angle (Hoffman, 1998; Sugihara, 2015b). As shown in Fig. 8, suppose that there is a convex surface, represented by the blue line, on a horizontal plane. We project this surface along the viewing direction 1 onto a horizontal plane, and get a picture. In Fig. 8, the projected images of the vertices are represented by black dots. Next suppose that we see this picture along the viewing direction 2 from the opposite side with the same downward-looking angle. Then, the picture matches another 3D structure that is obtained by reversing the heights of the original surface, as shown by the red line in Fig. 8.

The height-reversal property explains the visual effects in Fig. 2. The object in Fig. 2a is a plate with a non-zero thickness on which a convex surface featuring a mesh structure is drawn, but we perceive a concave surface in the mirror image. We use a plate with a certain thickness, instead of a thin material such as a single piece of paper, in order to make the concave surface seem more natural. The object in Fig. 2b is a square box on top of which a truncated pyramid is drawn, and we perceive a hole of the same shape in the mirror image. In this case also we use a box instead of a thin material in order to make the perception of the hole more natural.

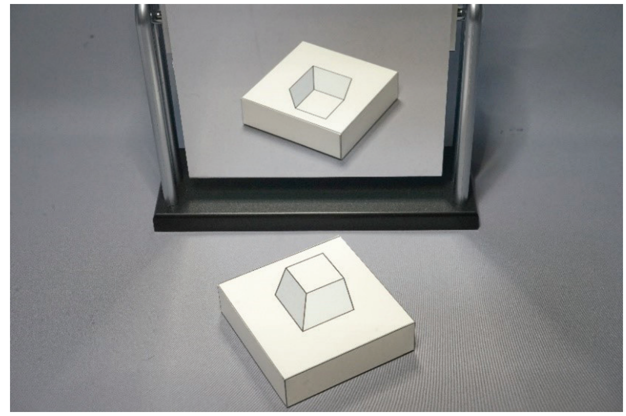

Fig. 9. Picture with an occlusion.

We should note that, if occlusion occurs, that is, if some part of the surface is hidden by other parts when it is seen from viewing direction 1, then the height reversal property is lost because the 3D interpretation becomes inconsistent when seen along the second viewing direction. Fig 9 shows an example in which the occlusion occurs. The truncated square pyramid drawn on the top face of the box in Fig. 2b is replaced with a steeper truncated pyramid. As the result of this, two side faces of the pyramid become invisible and the pyramid occludes part of the top face of the box. In this case, the mirror image does not make sense as a 3D object. Thus, the height-reversal illusion does not occur.

Lying-standing illusion

Fig. 10 shows how the lying-standing illusion occurs from the point of view of geometry. Suppose that we lay down a circular cylinder on a horizontal plane S. In Fig. 10, A1 represents the top circle of the cylinder and B1 represents the side of the cylinder. Projecting the cylinder onto S along a 45-degree downward direction, we get the picture P consisting of the top face A2 and the side face B2. At the bottom of Fig. 10, we show the projected picture seen from above. Next, we reflect the picture by the vertical mirror M, and get the mirror image Q consisting of the top face A3 and the side face B3. Finally, when we see this mirror image along the same viewing direction, we can interpret it as a standing cylinder with the top face A4 and the side face B4. Thus, the picture of a lying cylinder creates a mirror image corresponding to a standing cylinder.

This perceptual process works if: (1) the object has a subjectively perceived direction of the axis (such as the

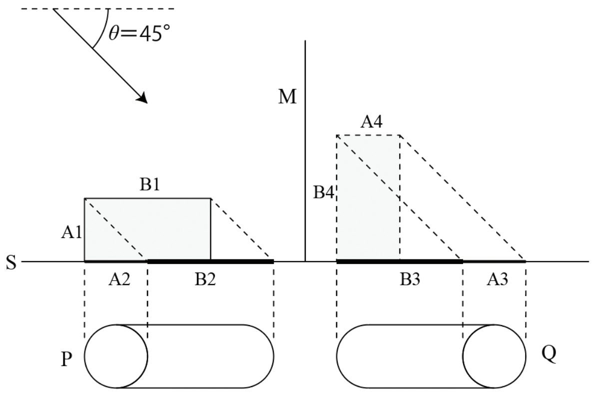

Fig. 10. Optical process of the lying-standing illusion.

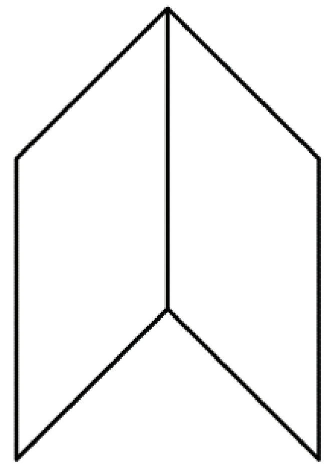

Fig. 11. Mach book.

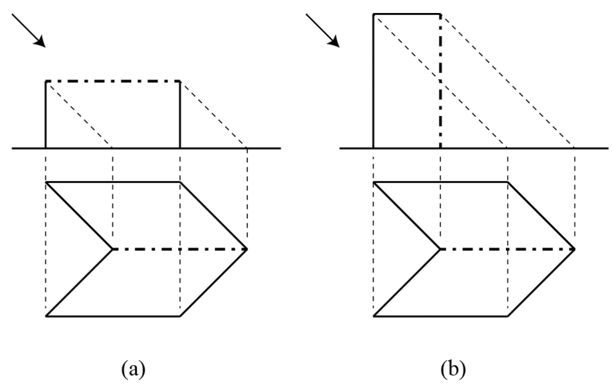

Fig. 12. Two possible interpretations of the Mach book: (a) a lying book, whose cover side is visible; (b) a standing book, whose inside pages are visible.

axis of a cylinder), (2) the axis is directed toward the viewer, and (3) the viewer sees the scene downward at 45 degrees. If the object does not have a clear direction of the axis or the axis is not directed toward the viewer, the mirror image cannot be interpreted as a standing counterpart of the original object. If the viewing angle is not equal to 45 degrees, the perceived object does not have the same height or the same section as the original object (Sugihara, 2022).

It might be interesting to compare the lying-standing illusion with the Mach book shown in Fig. 11 (Robinson, 1998). This picture has two interpretations; one is an open book laid down on a desk surface, and the other is an open book standing on a desk.

The source of ambiguity of the Mach book might be understood by Fig. 12, where the lower part presents the Mach book and the upper part represents the side view of the desk surface on which the posture of the book is perceived when the picture is seen from in the direction represented by the arrow. The centre vertical edge of the Mach book is drawn by the dash-dot line, and the associated 3D book edge is also represented by the same dash-dot line. Panel (a) shows the interpretation as a lying book, where the central vertical edge is interpreted as a convex edge, and panel (b) as a standing book, where the centre vertical edge is interpreted as a concave edge. Thus, the centre edge is interpreted as convex in one interpretation and as concave in the other interpretation.

Fig. 13, on the other hand, presents a lying-standing illusion created by a book, in which a book which is lying down flat stands in the mirror. The behaviours are similar, but there is a big difference. In the Mach book, the cover side of the book is shown in the lying-down position, while the inner-page side is shown in the standing position. In the rising book version of the lying-standing illusion, in contrast, the cover side is shown in both the lying-down position and the standing position. Therefore, the lying-standing illusion differs from the ambiguity of the Mach book.

Somersault illusion

The somersault illusion might be understood most easily among the five visual effects, because seeing a picture from the opposite side is almost equivalent to turning the picture upside down. The only differences are that we see the picture at a slanted direction instead of seeing it from exactly the front, and that we understand the direction of gravity from the position of the mirror.

The picture in Fig. 4a was drawn according to the rules of perspective projection. In particular, the horizon line is between the table top and the table foot, and hence the rear side of the table top and the upper side of the table foot are visible. This in turn results in the table top and the table foot being exchanged in the mirror, and we perceive the table to also be in a stable upright position in the mirror.

The picture in Fig. 4b was also drawn using perspective projection. This might strengthen the impression of the 3D structure, and we can perceive a plate seen from below although it appears to be unstable, as if it is suspended in the air. We chose the name 'somersault' for this type of illusion because of this figure.

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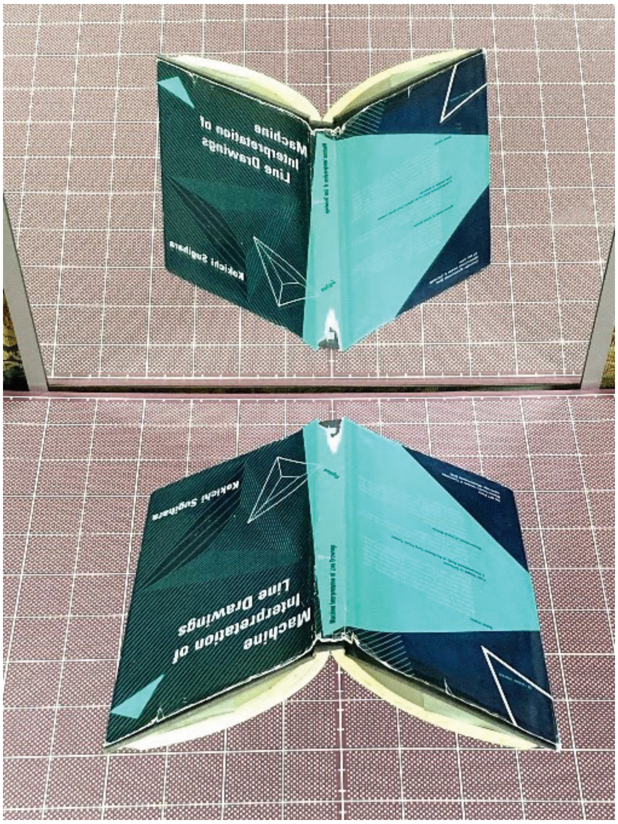

Fig. 13. Rising book.

Replacement illusion

The visual effect created by Fig. 5 can be explained geometrically by the height-reversal property. For Fig. 5a, the large upper flat area containing the red cone becomes the lower area in the mirror, and the small lower square area becomes the hill top in the mirror. Thus, the bottom and the top are exchanged in the mirror. The three intermediate steps of the staircases connecting the bottom and the top from the lowest to the highest also change their order from the highest to the lowest in the mirror. Thus, the relative heights of flat areas are reversed in the mirror. Also in Fig. 5b, if we see parts of the object one by one, we can construct a one-to-one correspondence between polygons constituting the object and their mirror images, and can understand that their heights are reversed.

Note that the pictures (excluding the red cones) in Fig. 5 have no occlusion. Hence, the height reversal relation between the object and the mirror image holds without inconsistency.

However, we intuitively feel that the object structure changes drastically instead of just reversing their heights. One reason might be the existence of vertical walls. Each staircase in Fig. 5a has two vertical walls, one of which bounds the volume below the staircases and the other representing a side wall that extends upward. However, the walls below the staircases become thin vertical plates and the walls extended upward become the walls below the staircases. These behaviours of the walls make it difficult for us to find the correspondence between the object and the mirror image, and consequently we feel that the object is replaced with another.

Note that the objects in Fig. 2 also have walls, that is, the side faces of the plate and the box. However, they are real walls and behave normally in the mirror. Hence, we only feel that the heights are reversed.

The objects in Fig. 5 also have the red cones. They behave normally as 3D objects in the mirror and they suggest the direction of gravity. This might also contribute to the sense of there being an anomaly.

Discussions

Although the objects are mainly 2D pictures in all five types of illusions, we perceive that they are 3D structures. This is mainly because we are looking at photographs instead of looking at the 3D scene (consisting of the picture and the mirror) directly. If we see the 3D scene, we can use our natural stereo vision and hence can recognize more easily that the objects are pictures. Seeing photographs, on the other hand, is equivalent to seeing the 3D scene with only a single eye because a camera has only one lens centre, and consequently we cannot use stereo vision. In addition, the pictures represent 3D objects mainly composed of rectangles and circles, which makes it easy for us to interpret 3D structures because human brains are apt to interpret parallelograms as slanted rectangles and ellipses as slanted circles (Gibson, 1950). If our brains were unable to perceive 3D structures, we would easily be able to understand the normal relation between a picture and its mirror image, and the illusions would not occur.

Another factor that makes us difficult to understand they are pictures is that each picture is cut along the boundary (i.e. along the silhouette) of the object instead of being bounded by a rectangular frame as an ordinary picture, and that it is placed in a 3D scene consisting of a desk surface and a mirror. An ordinary picture is bounded by a rectangular frame and consequently we understand that the picture represents a world which is separated by the frame from the real world in which we live. In our setting, on the other hand, there is no such separation frame, which strengthen the impression that we are looking at a real 3D object instead of a picture.

For all the five illusions, a picture is placed on a horizontal surface and it is reflected in a vertical mirror. These processes are identical. The difference only comes from how the mirror image is interpreted, in other words, the difference comes from the psychological behaviour of human perception. As a result of this, the classification is not necessarily stable. Indeed, the examples presented in Section 2 were chosen according to the author's subjective decisions.

For example, the object in Fig. 1b is classified as the left-right reversal. However, it can also be regarded as the result of exchanging the floor and the ceiling, and hence it can also be classified as a somersaulting object.

The object in Fig. 5a consists of two staircases. If we hide the left half of this object and see only the right half, it is almost the same as the object in Fig. 1a. If we hide the right half of Fig. 5a, we observe the situation in which the direct view and the mirror reflection of Fig. 1a are exchanged. So, the object in Fig. 5a can be considered as a combination of two left-right reversing objects.

The anomalous perceptions of objects in Figs. 1a, 2a, 2b, 5a and 5b can all be explained by the height-reversal property. Indeed, the convex edges change to concave and the concave edges change to convex when we move from the direct views of the objects to their mirror reflections. So, they all might be considered as replacement of one object to another. However, we perceive the left-right reversal in Fig. 1a; this might be provably because the picture admits 180-degree rotational symmetry. Similarly, we perceive the height reversal in Figs. 2a and 2b; this might be provably because the object represented by the picture admits 90-degree rotational symmetry. Therefore, an object belonging to the type of a replaced object might be classified to another type if it has some special properties such as symmetry.

From these examples, we have to say that the distinction is not very clear. It is a future research problem to investigate the relations among the five types of interpretations.

It is also interesting to observe that all five types of illusions occur not only when we do not recognize that the main part of the object is a picture drawn on a planar sheet, but also after we are told that it is a picture. It is well known that our brains cannot see a 2D picture purely as it is if the picture represents a 3D structure. The Shepard illusion shows that identical parallelograms, drawn in a picture as the top plates of desks with different postures, appear to be non-identical (Shepard, 1990). In the corridor illusion, two identical figures look different in size if they are located in a picture of a 3D scene, one at a near part and the other at a far part (Gibson, 1950; Richards & Miller, 1971). Therefore, even though we know that we are looking at a 2D picture, our perception is affected by the 3D structure suggested by the picture. The nature of our vision system inhibits us from seeing a picture and its mirror image as they are, and instead makes us see 3D structures and feel that the mirror image is not consistent.

The illusions listed in this paper come from ambiguity that the pictures have. However, the sense of impossibility is much stronger than with traditional ambiguous pictures. The Necker cube and Mach book, for example, create two interpretations, but only one interpretation occurs in our mind at each moment, and occasionally it is replaced with the other; the two interpretations do not exist at the same time. This is because the ambiguous pictures are viewed in an isolated manner. In our setting, the picture and the mirror image are seen at the same time. So, two interpretations, which could never occur at the same time if we see only the picture, exist in our mind. This setting may make us feel that the behaviour of the object and the mirror image is not possible.

One common property of the five types of illusions is the sense of impossibility. Each type creates one factor of impossible change, such as the left-right change, the height change and the posture change. We may augment the types of illusion by combining two or more factors.

One possible direction of the augmentation is to mix two types of illusions in a single picture. An example is show in Fig. 14. There are two lying-standing objects; a yellow lying cylinder rises us in the mirror, and a red standing cone lies in the mirror. Moreover, those objects are placed on a low flat but they are placed on a high flat in the mirror. Thus, this is a mixture of the lying-standing illusion and the replacement illusion.

Another possible direction of the augmentation is to mix a picture with a real 3D object. As we have already seen in Fig. 5, the addition of a real 3D cone makes the behaviour of the objects more complicated because the picture and the 3D object behave differently in the mirror. Fig. 15a shows another example in which a picture and a real 3D object are mixed. It seems that the triangular cylinder lying on a support box rises up in the mirror, but at the same time the support plate changes from rectangular to triangular. The fact is that the triangular cylinder is represented by a horizontal 2D picture, while the support plate is a real 3D pentagonal cylinder, as shown in Fig. 15b. The 2D picture appears to have risen up in the mirror while the 3D support plate behaves normally. As the result of this combination, the sense of inconsistency might be stronger than in the case of the simple lying-standing illusion object.

Concluding remarks

We have presented five types of visual effects created by the same optical process with a horizontally placed 2D picture and a vertical mirror. These might give new insights into the study of ambiguous pictures. Traditionally, ambiguous pictures are studied by viewing them from the front and sometimes by rotating the orientation. In the present paper, we view pictures from two directions simultaneously using a mirror, and so can compare two interpretations at the same time. Furthermore, we can view pictures in 3D environments including the desk surface on which the picture and the mirror are placed so that we understand the direction of the gravity. These factors may create new visual effects.

This is just a starting point to study ambiguous pictures using mirrors. The five types of illusions presented

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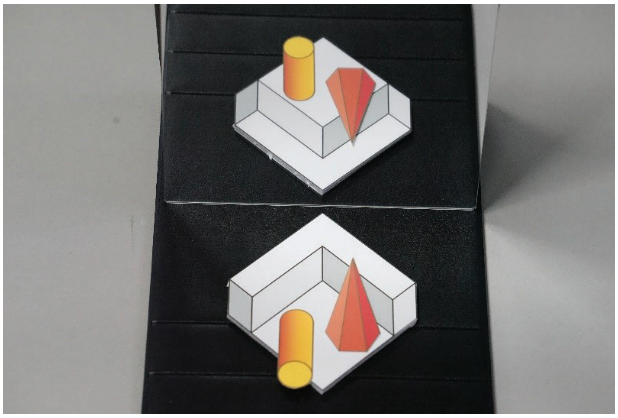

Fig. 14. Mixture of a replaced object and lying-standing objects.

(a) (b)

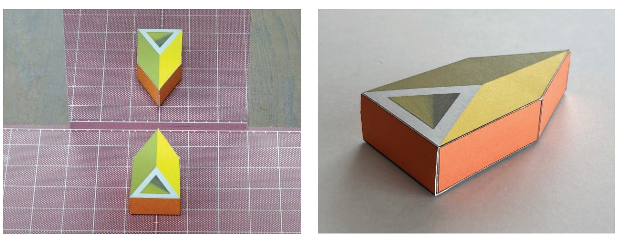

Fig. 15. Picture of a triangular cylinder augmented by a 3D plate.

here were found by the author accidentally. To search for still other types of illusions and to find some systematic method to exhaust the illusion types are left for future research.

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